The Effects of Proppant Concentration on the Rheology of Slurries for Hydraulic Fracturing – A Review

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A B S T R A C T

Hydraulic fracturing is a method employed by the petroleum industry to increase the performance of oil and gas reservoirs. Modeling this process requires accurate characterization of fluid rheology which is significantly affected by the proppant concentration. Various models for slurry viscosity and settling velocity have been reviewed. While many viscosity expressions exist, it was found that many may be inaccurate. The most valuable viscosity correlations are those derived from empirical data based on materials that are commonly used in the hydraulic fracturing industry.

F A C U L T Y  M E N T O R

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Nick has done a comprehensive study on the subject of hydraulic fracturing, which is a borehole stimulation technique used to increase the oil and gas production. His detailed study on the influence of the proppant concentration on the fluid rheology has provided very useful information for us and other people to develop more accurate computational models for hydraulic fracturing simulation. The experience I had with Nick completely changed my perception on undergraduate research. It is extremely gratifying for me to see an undergraduate student develop keen interests in a science and engineering subject, produce very useful results, and finally land gainful employment in the particular field in which he did the research.

A U T H O R

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Nicholas Petty is a graduating senior majoring in Mechanical Engineering with a concentration in Energy and Environment. Prior to returning to school to complete his engineering degree, Nicholas worked for Halliburton Energy Services, Inc. as an equipment operator on a hydraulic fracturing crew. He began researching hydraulic fracturing with Professor Guanshui Xu in the fall of 2010 and has since continued and expanded his petroleum studies to include operations, completions, artificial lift systems, and well performance analysis. Nicholas is a member of Toastmasters International and is the recent recipient of the Osher Reentry Scholarship. Nicholas would like to thank Professor Xu for his continual guidance and support.
INTRODUCTION

Annual crude oil production in the United States (U.S.) peaked in 1970 at 3.5 billion barrels and has since slowly declined. The late 1970s and early 1980s saw a brief increase in production; however, since 1985 each successive year has seen a decrease in production – that is, until 2009. As seen in Figure 1, 2009 saw the first significant increase in crude oil production in nearly twenty years [1].

![Figure 1: Annual Domestic Crude Oil Production](image)

Advances in technology have allowed engineers and geologists to locate new reservoirs. Also, advances in production enhancement techniques have allowed engineers to stimulate existing reservoirs and reservoirs that were originally deemed inaccessible. One such technique responsible for this increase in production is hydraulic fracturing, or fracking. The fracking process uses viscous fracturing fluid pumped at high pressure to fracture the rock formation under compressive stress. A proppant, such as sand, is added to the fluid and serves the purpose of keeping the fracture open. When pumping stops the fracture slowly closes on the slurry and the fluid seeps through the porous rock. The proppant that remains in the fracture acts as a conductive medium to facilitate the extraction of hydrocarbons.

While many types of materials can be used as a proppant, sand is one of the most common [2]. Sand is typically used when the net fracture closure stress is below 6000 psi. Stronger proppants, such as ceramics, are used when the net fracture closure stress is more than 6000 psi. Water is the most common base fluid. Powders, gels, and/or foams are usually added to the water in order to increase the fluid viscosity. One of the most common fluid additives is hydroxypropyl guar (HPG), a polymer powder [3]. When added to water, the powder hydrates and produces a gel. Other additives can be mixed with HPG to produce cross-linked gels for even greater viscosity.

The slurry viscosity determines fluid flow and proppant transport. Accurately determining the slurry viscosity is a challenging task; but it is also vitally important. If the fluid is not sufficiently viscous, the proppant will not be effectively transported throughout the fracture. If the slurry is too viscous, the fluid will not flow properly. By knowing the slurry viscosity, one can more accurately model and optimize the fracking process. The challenge in calculating slurry viscosity is complicated due to variables such as the addition of proppants to the fluid, and is also due to the multiple agents that may constitute the fracturing fluid.

RHEOLOGY

Rheology is the study of the flow of matter. Fluids can be characterized as being Newtonian or non-Newtonian. Newtonian fluids exhibit a linear relationship between an applied shear stress and the ensuing rate of deformation as seen in

\[ \tau = \mu \frac{du}{dy} \]  

(1)

where \( \tau \) is the shear stress, \( \mu \) is the viscosity, and \( \frac{du}{dy} \) is the rate of deformation [4]. For a Newtonian fluid, viscosity is constant; it does not vary with the rate of deformation. A non-Newtonian fluid can be described by the power law for fluids

\[ \tau = K \gamma^n \]  

(2)

where \( K \) is the flow consistency index, \( \gamma \) is the shear rate, and \( n \) is the flow behavior index (\( n = 1 \) for Newtonian fluids, \( n < 1 \) for pseudoplastic fluids, \( n > 1 \) for dilatant fluids). Equation 2 can be rewritten

\[ \tau = K\gamma^{n-1}\gamma \]  

(3)
where the term $K_{\gamma}^{n-1}$ is referred to as the apparent or effective viscosity ($\mu_{\text{eff}}$). This modification allows the power law to be in the same form as Equation 1 with

$$\tau = \mu_{\text{eff}} \gamma \quad (4)$$

**Newtonian Expressions**

The following is a review of expressions used to determine the viscosity of Newtonian fluids with particles and the viscosity of non-Newtonian fluids with particles. Most hydraulic fracturing fluids are non-Newtonian. Nevertheless, a brief historical review is presented of those that have investigated the effects of particle concentration on viscosity.

In 1905, Albert Einstein submitted a doctoral thesis to the University of Zurich [5]. In the dissertation Einstein used experimental data to prove Avogadro’s number and the radius of a molecule. To achieve his objective, Einstein proposed a theoretical model for determining the viscosity of a slurry, sugar water in this case, based on the viscosity of the clean fluid and the volumetric concentration of particles. Determining Avogadro’s number and the radius of a molecule required other equations that were functions of viscosity. To determine the viscosity, Einstein proposed

$$\mu = \mu_0 (1 + 2.5c) \quad (5)$$

where $\mu$ is viscosity of the slurry, $\mu_0$ is the viscosity of the clean Newtonian fluid, and $c$ is the volume fraction of particles. Experimental analysis has shown that this expression is only accurate for very dilute slurries.

In 1941, Eilers [6] proposed an expression that used the maximum volumetric concentration of particles with

$$\mu = \mu_0 \left(1 + \frac{1.25c}{1-c/c_{\text{max}}}\right)^2 \quad (6)$$

The work of Frankel and Acrivos in 1967 showed that once the particle concentration reached 80% of the maximum concentration, the determination of the viscosity must incorporate the ratio of the particle concentration to the maximum particle concentration [7]. This is seen in the Frankel-Acrivos equation

$$\mu = 1.125\mu_0 \left[\frac{(c/c_{\text{max}})^{\frac{1}{3}}}{1-(c/c_{\text{max}})^{\frac{1}{3}}}\right] \quad (7)$$

where $c/c_{\text{max}}$ is the volumetric concentration of solids normalized to the maximum solids concentration.

Another accepted viscosity expression is the Landel equation from 1963 [8]. For particles, Landel and his colleagues used glass beads, metal powders, and ammonium perchlorate with diameters approximately ranging from 10 to 100 microns. Landel’s empirical equation is

$$\mu = \mu_0 \left(1 - \frac{c}{c_{\text{max}}}\right)^{-2.5} \quad (8)$$

The prediction of the relative viscosity for each equation can be seen in Figure 2. The expressions agree at lower particle concentrations and diverge at higher concentrations.

**Modifying Newtonian Expressions**

When proppant is added to a fluid, determining the slurry viscosity may be possible by modifying an existing Newtonian equation. For example, a viscosity multiplier, $m_{\mu}$, can be derived by

$$m_{\mu} = \frac{\tau_p}{\tau_0} \quad (9)$$

where $\tau_p$ is the shear stress with proppants and $\tau_0$ is the shear stress without proppants. Nolte modified the Landel equation (6) for power law fluids resulting in
where \( n \) is the flow behavior index of the non-Newtonian fluid without proppant [9]. Nolte has shown that Equation 10 is accurate for low concentrations of proppant. Equation 10 is limited because it uses the flow behavior index of the non-Newtonian fluid without proppant and because it does not consider shear effects. Many researchers, including Nicodemo and Acrivos, concluded long ago that viscosity is dependent upon shear rate, but chose not to consider it for the sake of simplicity.

Keck et al. incorporated the effects of shear rate in modifying the Eilers equation (6) with

\[
\mu = \mu_0 \left( 1 + f(n, \gamma) \right)^{1.25c} \left( \frac{1.5c}{\gamma} \right)^{1.5c}
\]

(11)

where \( f(n, \gamma) \) is a function of \( n \) and \( \gamma \) for a fluid without proppants [10]. To produce \( f \), Keck applied three boundary conditions to \( n \) and \( \gamma \). This gives the final modification of Eiler’s equation with

\[
\mu = \mu_0 \left( 1 + \left[ 0.75 e^{1.5c} - 1 \right] e^{-\frac{\gamma(1-n)}{1000}} \right)^{1.25c}
\]

(12)

where \( n \) and \( \gamma \) are properties of the non-Newtonian fluid (base gel) without proppants and \( c_{\text{max}} = 0.66 \). By employing \( n \) and \( \gamma \) of the base gel, Keck’s modification of the Eilers equation considers the effects of temperature and shear rate. Keck has shown that this modification accurately predicts slurry viscosity for HPG gels and 60/100 mesh styrene divinylbenzene (SDVB) beads.

**EMPIRICAL DETERMINATION OF VISCOSITY**

Shah gathered empirical values of relative viscosity. Shah chose not to measure viscosity using existing Newtonian expressions and a modified viscometer as his predecessors had done. Instead, Shah used a vertical fracture apparatus to determine relative viscosity. By doing so, Shah was able to consider the effects of temperature, shear rate, gel concentration, and sand concentration [11]. The vertical fracture apparatus consisted of two large sheets of Plexiglas® with a known width \( (w) \) and length \( (L) \). By pumping slurry between the Plexiglas® sheets and monitoring the differential pressure, \( \Delta p \), and slurry velocity, \( V \), Shah was able to determine the relative slurry viscosity with

\[
\frac{w \Delta p}{2L} = k \left( \frac{6V}{w} \right)^n
\]

(13)

A regression analysis of Shah’s findings gives

\[
\mu_s = Ae^{Bc}
\]

(14)

and

\[
K_s = Pe^{Qc}
\]

(15)

where \( A, B, P, \) and \( Q \) are experimentally determined constants. Plots of \( K_s \) and \( n_s \) can be seen below in Figure 3 [12].

Shah’s results were compared to Nolte’s and Keck’s predictions. Shah concluded that Keck’s model was more consistent with experimental results than Nolte’s model. Nolte’s model was most accurate at low sand concentrations, but consistently predicted lower values of relative viscosity. Keck’s model provided more accurate predictions of relative viscosity, but the predictions were slightly higher than those that were experimentally determined. The inaccuracy may be due to the different proppants (Keck used SDVB beads, whereas Shah used sand) or perhaps the inaccuracy is a manifestation of
the challenges in modifying an equation not suited to characterize this type of fluid.

**REVIEW OF PRESENTED Viscosity EXPRESSIONS**

Up to this point, determination of a slurry’s viscosity has been approached in four ways:

1. \( \mu = f(\mu_0, c) \)  
2. \( \mu = f(\mu_0, c, c_{\text{max}}) \)  
3. \( \mu = f(\mu_0, c, c_{\text{max}}, n) \)  
4. \( \mu_{\text{eff}} = f(\mu_0, c, c_{\text{max}}, n, \gamma) \)

Methods 3 and 4 are modifications of method 2 and, in each case, \( n \) was assumed to remain constant with the addition of proppants. However, addition of proppants will increase the apparent viscosity of a base gel, and the assumption that \( n \) remains constant may be inaccurate. Shah has shown that \( n \neq n_s \) and \( K \neq K_s \), where \( n_s \) and \( K_s \) are the power law fluid parameters for a fluid with solid loading [11]. As the concentration of proppants increases, \( n_s \) decreases and \( K_s \) increases. And yet, as temperature increases, \( n_s \) increases and \( K_s \) decreases. In light of these realities, it may be more practical to describe a power law fluid using power law parameters that have been empirically determined than to modify existing Newtonian equations.

**TERMINAL VELOCITY**

The proppant reaches its terminal velocity once the buoyancy and drag forces reach equilibrium with the gravitational force. As a fracture forms, the proppant should ideally be uniformly dispersed throughout the fracture. However, the force of gravity acts on the proppant pulling it down and may cause it to settle prematurely. Therefore, the settling velocity is an important parameter used to predict proppant transport. In 1963, Thomas developed an empirical expression to account for a concentration of particles with

\[
2.303 \ln \frac{V_c}{V_0} = -5.9c
\]

where \( V_c \) is the settling velocity of particles in a concentration [13]. For this expression, Thomas used alumina and graphite, as well as other materials. The particles had a diameter range of 0.4-17 microns, considerably smaller than the 420-841 microns of 20/40 mesh sand. It should also be noted that this expression is for a system with no fluid motion.

**SUMMARY AND CONCLUSION**

The fracking process requires accurate characterization of fluid rheology, which is affected considerably by the proppant concentration. Errors in modeling this process can result in less than optimal well performance at a significant monetary cost. A review of the effects of proppant concentrations on the rheology of hydraulic fracturing fluids has been presented. The literature shows that many expressions for slurry viscosity exist. Some expressions were derived theoretically and others empirically, while others were developed by modifying existing equations. Hydraulic fracturing can be a specialized undertaking; each slurry may be unique in its composition of chemicals and proppants. Therefore, an all-encompassing viscosity...
expression which takes into account temperature, gel concentration, and proppant concentration and size will not be accurately determined by modifying existing equations that were never intended to account for so many parameters. It is evident that empirical data that relies on materials that are commonly used in frac jobs will provide the most valuable correlations. This review has considered the effects of proppant concentration on proppant transport. Other parameters that should be considered include fracture width and fluid elasticity [15, 16].

REFERENCES